The Influence of the Number of Satellites on the Accuracy of RTK GPS Positions

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Abstract

The aim of this paper is to analyse the effect of the number of satellites on the accuracy of RTK positions. Testing was undertaken on a ten kilometre baseline with measurements collected under varying satellite configurations over a four day period. Approximately three thousand positions were recorded. The results indicate that an increase in the number of satellites has a limited influence on position accuracy in an urban environment.

Introduction

The Global Positioning System (GPS) has become a widely used tool for a number of positioning applications. The two GPS carrier signals, L1 and L2, are used extensively for cadastral, topographic and engineering survey applications. The use of the carriers requires the identification of the integer cycle ambiguity inherent in the phase measurement of the carrier signal. Once the integer ambiguities are identified and constrained, the position of a roving receiver can be estimated to an accuracy of, generally, better than two centimetres with respect to a stationary reference receiver. The application of a reference and roving receiver is referred to as differential positioning. There are a number of errors affecting GPS observations that are removed by the differential technique. The residual errors remaining in position estimates are primarily due to multipath, orbital errors and unmodelled atmospheric errors. It is these influences that limit the use of GPS for high precision applications. For kinematic GPS applications, occupation periods are typically less than one minute, thus limiting the averaging of residual errors.

The use of a data link, to transfer measurements acquired at the reference receiver to the roving receiver, permits the calculation of the rover coordinates at the time of measurement. This survey technique is termed Real Time Kinematic (RTK) and is continuing the revolution that GPS is having on survey practice. The development of RTK enables surveyors to coordinate marks of interest in a rapid and efficient manner. The real time capability enables checks on computed positions to be performed in the field, a requirement of cadastral survey legislation in many states and countries [Boey and Parker 1996]. The technique also facilitates the placing of marks at pre-determined coordinates.

The GPS constellation is designed to support continuous tracking of a minimum of four satellites at any location on the Earth. In many instances, up to eight and nine satellites may be visible at certain times of the day. The number of tracked satellites has a large influence on the strength of the satellite configuration for positioning, measured using the Dilution Of Precision (DOP) factor. The aim of this paper is to assess whether the number of satellites effects the accuracy of RTK positions.

The RTK observation procedure is reviewed, as are the satellite coverage characteristics of the GPS constellation. This is followed by the description of a field test designed to enable the influence of the number of satellites on position accuracy to be evaluated.

Real Time Kinematic Positioning

Precise positioning using GPS measurements requires the measurement and processing of the L1 and/or L2 carrier signals. The phase of the satellite carriers can be measured to a few millimetres by almost all commercially available GPS receivers [Leick 1995]. The Cartesian coordinate difference, or *baseline*, between a stationary reference receiver and other roving receivers can be computed to an accuracy suitable for many surveying tasks if the integer cycle ambiguity of the carrier phases can be correctly determined and constrained. Ambiguity resolution techniques, such as known baseline occupations and on-the-fly resolution, in particular, are extremely effective in rapidly identifying the ambiguities [Hatch 1991, Landau and Euler 1992]. The on-the-fly technique has the advantage of being able to operate successfully while the roving receiver is in motion. In most surveying applications, the position of the receiver while in motion is not of interest. Regardless of this, the most efficient operation occurs when the receiver continuously tracks at least four satellites (and preferably five or six) for the duration of the survey. This observation technique, stop and go *kinematic*, enables the surveyor to occupy points of interest for only a few seconds as the integer ambiguities have already been determined and are maintained while the surveyor moves between points.

In the 1980's and early 1990's, results from all GPS surveys were only available after the survey had been completed and the data *post-processed*. Post-processing provides robust baseline estimates as all measurements can be manipulated a number of times using least squares estimation techniques. The restrictions of post-processing from the surveyors perspective are that field checks and set-outs cannot be performed. Real time kinematic (RTK) GPS surveying introduces a mechanism for transferring the measurements acquired at the reference receiver to the roving receiver as soon as they are collected. This transfer mechanism, termed the communications link, is usually performed by a form of radio modem. The roving receiver processes the measurements from both receivers and displays the computed position information to the user in the field. As the position of the roving receiver is required in a timely manner, there is limited time for the rover to pass through previous measurements. Therefore, real time surveying is less robust than post-processing, however, the accuracy and precision attainable is still suitable for a large number of surveying applications.

Figure 1 presents a graphical overview of RTK surveying.

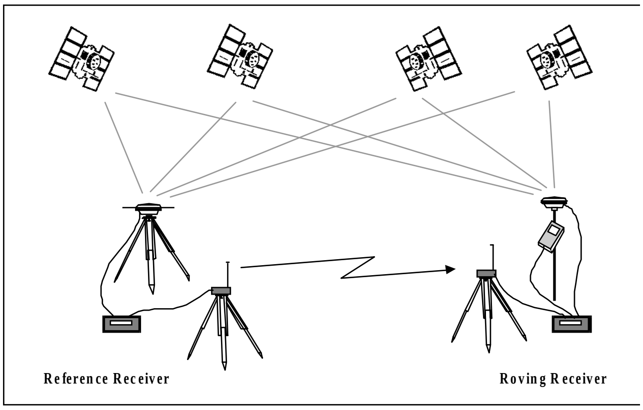

Figure 1. *Real Time Kinematic Equipment Configuration*

Real time processing at the roving receiver provides Cartesian coordinates, computed relative to the reference station coordinates. As the reference station coordinates are not always known, the coordinates of the roving receiver are generally presented as a three-dimensional Cartesian coordinate difference from the reference station. Coordinates of points are then propagated using these baselines and knowledge of at least one point with known coordinates.

In many countries, the transfer of data from the reference receiver to the rover is regulated by State and Federal communication agencies. To preserve frequency allocation, there are frequency and power restrictions which regulate the use of such communication devices. To avoid licensing of radios, low power radios are often used as the output signal is not considered strong enough, by regulation, to cause interference with other signal transmissions. The outcome of this approach is that users are restricted by these radios which often require line of sight operation and a limited range. Although repeater radios can be used to propagate transmissions, most RTK surveys have been performed over small areas ranging from a few hundred metres to, typically, less than five kilometres [Boey et al. 1996, Griffioen et al. 1993]. With the development of permanent tracking GPS receiver networks [Takac and Hale 1996, NRE 1998], users will expect to survey in real time over significantly larger distances than currently surveyed. The limits of RTK positioning are generally considered to be in the range of ten to fifteen kilometres, however, there is little information available in the literature that defines the level of performance that surveyors can expect under these conditions.

Satellite Coverage

The GPS constellation is designed to provide continuous satellite tracking to users anywhere on, or near, the surface of the Earth. This is accomplished using 24 satellites orbiting in six inclined planes located at a nominal altitude of 20,200 kilometres. This enables at least four satellites to be continuously visible anywhere on the Earth, the minimum number required for three-dimensional positioning.

In most locations on the Earth, five or six satellites are generally visible. This has a number of advantages for RTK positioning. The number and geometric spacing of visible satellites affect the time required to resolve the integer cycle ambiguities. The greater the number of satellites, the quicker the resolution of the ambiguities in most circumstances [Allison et al. 1994]. In addition, when more than four satellites are tracked, the redundant satellites can be used to detect erroneous measurements, facilitating more robust solutions. The instantaneous position of the satellites at the time of measurement will also directly influence the manner in which any measurement errors are distributed into the final computed position estimate. The strength of the satellite constellation at any point in time is indicated by the Position Dilution Of Precision (PDOP) factor. When satellites are spaced closely, PDOP values are large and position errors do not tend to cancel. This is illustrated by Figure 2.

The test described in the following sections assesses the impact of the satellite constellation over a ten kilometre baseline.

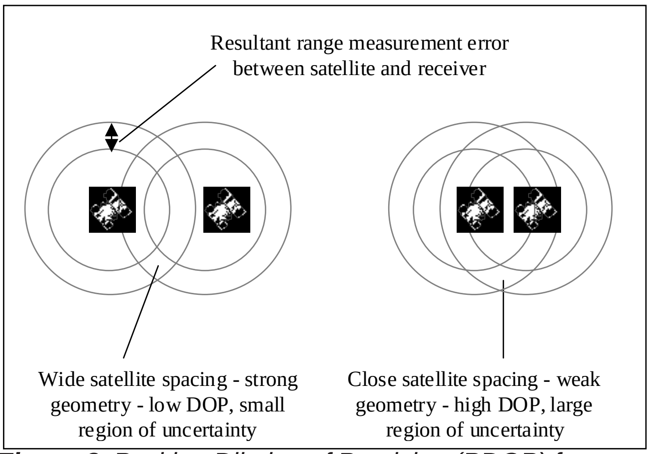

Figure 2. *Position Dilution of Precision (PDOP) factor*

Test Description

To evaluate the influence of the number of satellites and satellite geometry on RTK position accuracy, a computer program was written to control the operation of a roving GPS receiver. The primary function of the program is to extract RTK position information, including the number of satellites and PDOP values before forcing the receiver to temporarily stop tracking the satellites, thereby initiating a new ambiguity search and eventually a new set of resolved ambiguities. Each position estimate can then be considered independent for further analysis.

The program is controlled by a timer which polls the receiver for its best available position estimate at fifteen second intervals. The primary motivation for selecting fifteen seconds as a polling rate is due to traditional survey expectations that suggest that position estimates should be available in periods of less than fifteen seconds. If the carrier phase ambiguities have been resolved, the roving latitude, longitude and height are written to a text file. The receiver is then forced to temporarily stop tracking the satellites. This causes the receiver to re-acquire the satellites. If the receiver is polled and the ambiguities are not resolved, the output file is not modified. Figure 3 presents a flowchart of the operation of the receiver controlling program.

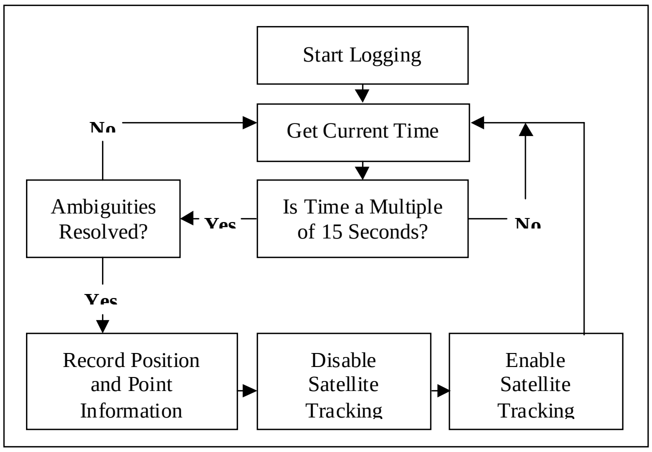

Figure 3. *Flowchart of computer program operation*

The program is currently capable of interfacing with the Trimble 4000SSE receiver. This model was initially released in 1993 and has since been superseded by new receivers with more advanced firmware containing updated ambiguity resolution and error modeling techniques [Trimble Navigation 1996, Trimble Navigation 1998]. Regardless, there are many receivers of this era in use in Australia, thus, analysis of the performance of this particular model will provide a benchmark for these users.

The manufacturer specifications for the Trimble 4000SSE receiver are ±(1cm+2ppm) for the horizontal component and ±(2cm+2ppm) for the vertical component. An additional recommendation includes a maximum range specification for operation of ten kilometres. The length of ten kilometres was selected in the testing to evaluate the accuracy obtainable at the extent of the manufacturer specifications.

The reference receiver was located on the RMIT astronomy observation deck in Melbourne. The roving receiver and notebook computer running the controller software were located at Essendon airport, ten kilometres North West of Melbourne. VHF radios were used to transmit the reference station data to the rover in real time. Observations were collected for a period of approximately 24 hours on each of the four days. In total, almost three thousand initialisations were performed, providing a large test sample of independent position estimates.

A 24 hour static survey was performed and processed to provide a basis for comparing the RTK observations. The International GPS Service for Geodynamics (IGS) precise ephemeris was used during static data processing. Differences in east, north and height components were tabulated and summarised for further analysis.

Throughout the testing, a number of rogue positions or bad initialisations were detected, resulting from the incorrect determination of the integer cycle ambiguities. For the purposes of this analysis, a bad initialisation was considered one where the coordinates of the point varied from the static solution by more than three times the manufacturer's specification for baseline component standard deviation.

In almost all instances, these occupations clearly exceeded this tolerance, in some cases, by several metres. In total, the success of the ambiguity resolution process exceeded 98 percent. The majority of failures occurred when five and six satellites were tracked. Interestingly, more than half of the bad initialisations occurred on the first day of the test. While satellite geometry appears to be a governing factor in the initialisation failures, the results of the first day were not repeated on the successive days of observation. Radio Frequency (RF) transmissions were suspected to be the influence as the roving receiver was located in an environment that is subject to a high amount of RF transmissions. However, investigations at Essendon and RMIT revealed that there were no RF anomalies during the recording. Other possible causes for the resultant bad initialisations are still under investigation.

If the results of the first day are ignored, the ambiguity resolution reliability exceeds 99 percent. In practice, observation of several minutes of stationary measurements are required to enable the satellites to move with respect to the receiver and facilitate detection of incorrect ambiguities [Allison et al. 1994]. In many instances, a number of points may have been surveyed before the user is notified of this occurrence. Therefore, it is essential that bad initialisations are detected and corrected as soon as possible. In all subsequent analysis, position estimates affected by bad initialisations have been removed.

Results and Discussion

Table 1 presents a summary of the data collected over the four day test period. Included is the number of initialisations, bad initialisations and the elapsed time period.

The observations on the ten kilometre baseline were collected at different times of day, therefore, the satellite configuration conditions were different for each of the four survey days. To provide a meaningful statistical analysis, the measurements were pooled and then divided based on the number of satellites used to calculate each sample position. The observation summary is presented in Table 2. The average PDOP values and initialisation times are also shown.

To graphically ascertain the distribution of the computed errors, derived using the static survey solution as the basis for comparison, the pooled sample was filtered for bad initialisations and a histogram of errors generated for the east, north and height components (Figure 4).

From the results presented in the histogram, it is evident that the east coordinate component is estimated the most precisely. There is a small offset from the mean of the samples, 8mm to the east. The north component also appears normally distributed about the mean, however, is less precise than the east component and is biased to the south by 12mm. Of most concern is the bias in the height component.

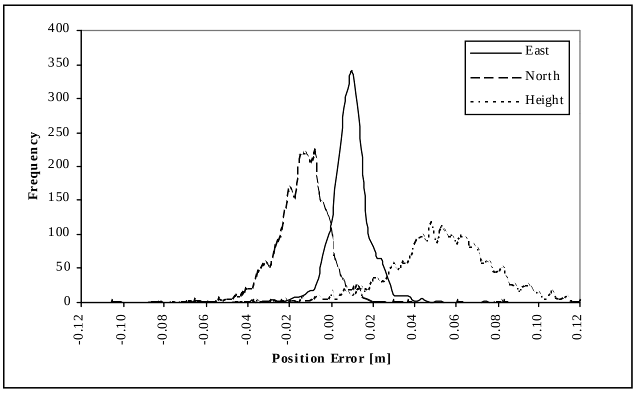

Figure 4. *Histogram of errors for all correctly resolved samples*

Although the precision of the height estimates is less than the horizontal components, this is expected due to the inherent nature of GPS positioning. The 50mm bias evident by the histogram indicates a possible incompatibility between the ambiguity resolution process used to derive the RTK positions and the static solution. As the majority of the bias is felt in the height component, this tends to suggest a possible discrepancy between the atmospheric models used during the resolution process. The L1 carrier phase solution was used for both the RTK and static solution types, therefore modeling errors of the troposphere and ionosphere may be a potential cause of this bias. The authors are currently investigating the reasons for this bias with the manufacturer.

Day of Survey	Date of Survey	Number of Initialisations	Number of Bad Initialisations	Elapsed Time (hours)
1	6-7th March 1998	645	27 / 4.2%	25.0
2	10-11th March 1998	838	5 / 0.6%	23.8
3	11-12th March 1998	811	6 / 0.7%	24.2
4	12-13th March 1998	645	11 / 1.7%	21.5
Totals		2939	49 / 1.7%	94.5

Table 1. Summary of observations

Number of Satellites	Number of Initialisations	% of Total Initialisations	Number of Bad Initialisations	Average PDOP	Average Time to Initialise (secs)
5	184	6.3	6 / 3.3%	4.2	166
6	1015	34.5	31 / 3.1%	2.8	117
7	1201	40.9	10 / 0.8%	2.3	79
8	445	15.1	2 / 0.4%	1.9	79
9	94	3.2	0 / 0.0%	1.7	91
Totals	2939	100.0	49 / 1.7%	2.5	98

Table 2. Summary of pooled observations

Regardless of the biases between the mean of the RTK samples and static solution, the results generally agree with the specifications provided by the manufacturer. The precision of the results indicates performance of one to two parts per million of the baseline length. In addition, due to the architecture of the controlling program, a maximum averaging period once the ambiguities have been resolved is fifteen seconds. In many cases, the averaging period will be less than this with a mean value most likely to be in the order of seven or eight seconds. If the polling period was increased to, say, one minute, it can be anticipated that the results would indicate an increase in precision [Frei and Beutler 1990]. A summary of the errors produced during the test is presented in Table 3.

From the information presented, it is evident that there is a strong similarity between the east, north and height errors when different satellite numbers are tracked. The previously mentioned biases in the mean values portayed by Figure 4 are particularly notable. There is a large range in the errors for almost all of the components, with the height component ranging by more than 0.26 metres when six satellites are observed. A more visual approach to analysing the mean and standard deviation values is obtained by graphing the error values against the number of satellites. Figure 5 presents the mean error and Figure 6 the standard deviation, versus the number of satellites.

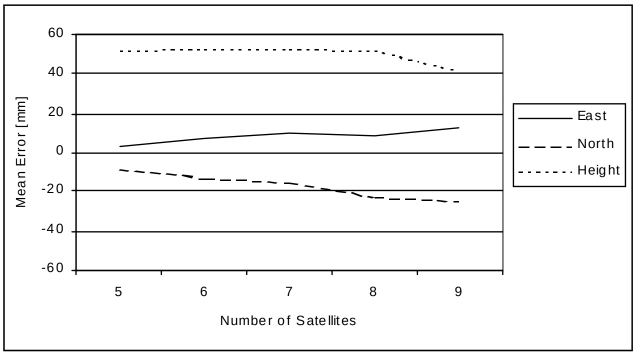

Figure 5. *Mean Error versus Number of Satellites*

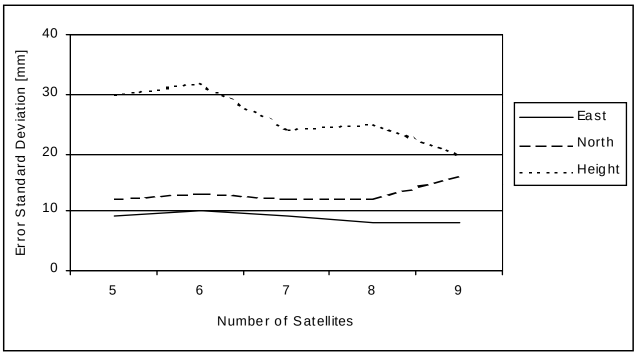

Figure 6. *Error Standard Deviation versus Number of Satellites*

The mean error shows little correlation between the errors obtained and the number of satellites tracked. In relation to the east and north components there is no significant improvement in the accuracy or precision of the observations as the number of satellites increase. The only noticeable variation is in the height component, where the standard deviation does reduce as the number of satellites increase. Although the height does show an improvement, the negligible change in accuracy or precision suggests that the number of satellites is not a contributing factor. A further appreciation on the effect of the number of satellites may be gained by analysing the effects of satellite geometry.

The Position Dilution of Precision (PDOP) values are used to represent the precision of an observation based upon the resultant satellite geometry. The average PDOP values from Table 2 indicate that the effect of satellite geometry on the accuracy of the positions is reduced as the number of satellites increase. Figure 7 however, relates the positional accuracy obtained to the PDOP values recorded during the observation sessions for two satellite configurations.

			Error Summary (mm)				
Satellites	*Samples*	*Coordinate*	*Mean*	*Std. Dev.*	*Median*	*Max. +ve*	*Max. -ve*
		East	4	9	4	34	-19
5	178	*North*	-9	12	-9	33	-47
		Height	52	30	56	121	-46
		East	7	10	7	74	-40
6	984	*North*	-13	13	-12	83	-105
		Height	53	32	53	184	-83
		East	10	9	9	50	-22
7	1191	*North*	-15	12	-13	80	-86
		Height	53	24	52	172	-29
		East	9	8	9	73	-12
8	443	*North*	-23	12	-22	8	-77
		Height	52	25	53	114	-36
		East	13	8	12	37	-5
9	94	*North*	-25	16	-21	-1	-70
		Height	42	20	43	84	-15

Table 3. *Summary of position errors*

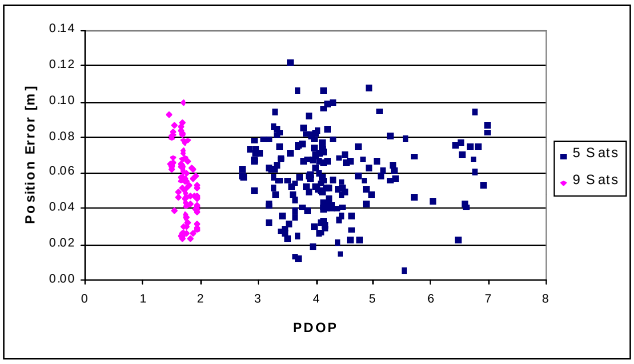

Figure 7. *Position Error in Relation to PDOP*

In contrast to the PDOP definition, Figure 7 indicates that the observations with large PDOP values (5 sats) exhibit precision values that compare with those positions determined with low PDOP values (9 sats). The relationship suggests that the PDOP values hold no significant relationship with the positional accuracy of the RTK test. A further appreciation of the effect of satellite geometry on the position determinations can be gained by examining two distinct satellite configurations.

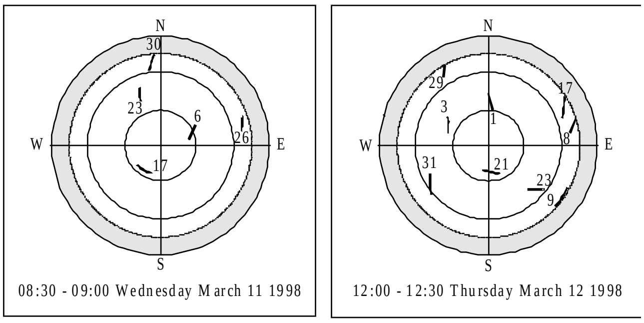

Figure 8. *Skyplot of Five Satellites (left) and Nine Satellites (right)*

The skyplots shown in Figure 8 represent typical satellite geometry configurations during the recording of the RTK observations.

The two cases examined include position determinations undertaken with five satellites and a large PDOP value compared to nine satellites with a low PDOP value. In the case of five satellites being visible in the observation window (PDOP 3.6), three of the satellites are positioned at high elevations. When nine satellites are being tracked (PDOP 1.9) four are also situated at a high elevation, however in this case five lie at lower elevations. Satellites at low elevations generally contribute an increase in the strength of multipath interference due to the reduction in the angle of reflection [Leick 1995]. In the case of nine satellites being tracked, over half are situated at low elevations, increasing the possible effects of multipath.

The two sites chosen for the RTK test include a number of reflective objects that result in the observations being susceptible to the effects of multipath. While the urban environment is not an ideal test site, due to the reflective surfaces, the purposes of the tests were to examine the accuracy of RTK positions under practical conditions. From the results presented it is evident that the accuracy and precision of the RTK observations show no significant improvement due to an increase in satellite numbers. However, due to the environment of the observations, potential benefits associated with the increase in satellites are removed due to the increased effects of multipath. For an association to be made between satellite numbers and accuracy of positions, further testing would be required in an ideal controlled environment. One consideration regarding the increase in satellite numbers is that the majority of bad initialisations occurred when five or six satellites were tracked. Consequently, the correct identification of the integer ambiguities is improved by the increase in the number of satellites tracked. While the accuracy may not be improved in an environment susceptible to multipath, the reliability is improved by the correct identification of the ambiguity.

Conclusion

The aim of this paper was to analyse the influence of the number of satellites on the accuracy of Real Time Kinematic GPS positions. To provide a large set of RTK positions, a program was designed to control the roving receiver at the extents of the systems operation. Approximately three thousand positions were collected over the four day observation period.

The results indicated that an increase in satellites made no significant contribution to the accuracy of the RTK positions, although the reliability of the ambiguity resolution process did improve. While this paper represents results obtained in real world conditions, further research in varying environments is required to provide a true assessment of the influence of the number of satellites on RTK positions.

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